Modelling contiguity in spatial decision contexts

V. Brison, M. Pirlot

UMONS - Faculty of Engineering

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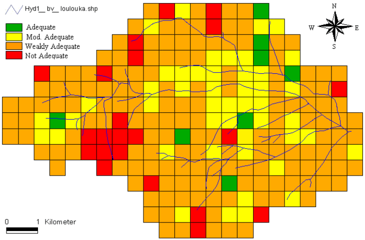
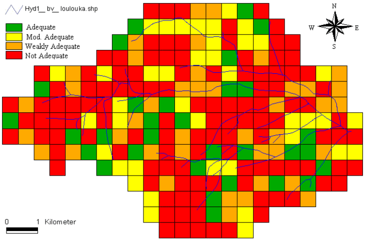
We want to know which map is better

Response to the risk of degradation of the landscape
(Loulouka Basin, Burkina Faso)



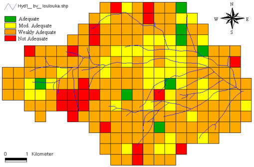
Previous state

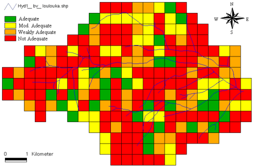


New state

Evolution for the better or the worse?
Need of a model for comparing decision maps

Problem 1


Hyd1__bv__loulouka.shp
Adequate
Mod. Adequate
Weakly Adequate
Not Adequate
N
W
E
S
0
1 Kilometer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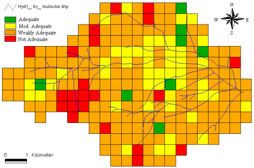
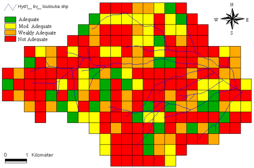
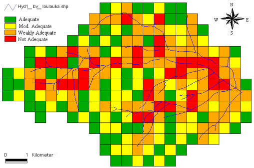
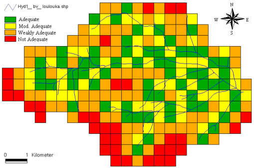
Problem 1





Problem 2





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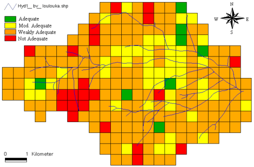
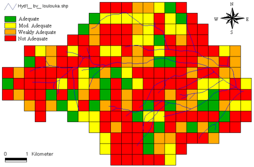
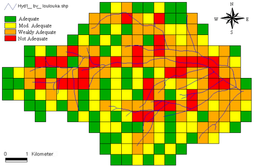
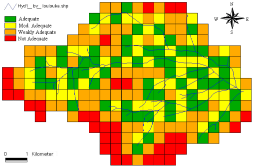


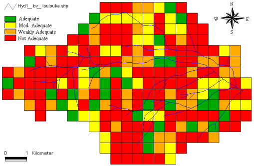


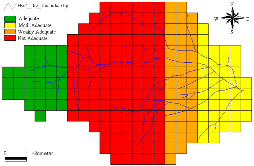
Problem 2





Problem 3





Comparing maps is a multi-criteria problem

S: territory composed of pixels

$s \in S$: pixel

$\gamma(s)$: evaluation of s on a common scale for all pixels

$\mathcal{C} = \{C_1, \ldots, C_n\}$: the common evaluation scale, composed of n categories

$A = (S, \gamma)$: evaluated geographic map (= decision map)

$\mathcal{A}$: set of evaluated maps

$\succsim$: preference relation on $\mathcal{A}$

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Comparison with a MCDA problem

maps $=$ alternatives

evaluations associated with the pixels $=$ criteria

Remark: common evaluation scale for all pixels $\Rightarrow$ commensurable scales

1. Comparing maps in a basic way
2. Taking geographic aspects into account
3. Taking contiguity into account

Model 1:

Comparing maps in a basic way

We use an expected utility model

Expected utility model

$A = (S, \gamma), B = (S, \delta)$

$$A \succsim B \iff \frac{1}{N} \sum_{s \in S} u(\gamma(s)) \geq \frac{1}{N} \sum_{s \in S} u(\delta(s)),$$

with $\gamma(s) \in \{C_1, \ldots, C_n\}$ and N is the total number of pixels

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Rewriting the expected utility model

$u(\gamma(s)) = u_i$ if $\gamma(s) = C_i$

$x_i(A) = \frac{|\{s \in S : \gamma(s) = C_i\}|}{N}$

$$A \succsim B \iff \sum_{i=1}^{n} x_i(A) u_i \geq \sum_{i=1}^{n} x_i(B) u_i$$

The model assumes that the region is homogeneous

Hypothesis

We assume that the whole region is *homogeneous*:

$$x(A) = x(B) \Rightarrow A \sim B$$

with $x(A) = (x_1(A), x_2(A), \ldots, x_n(A))$ the area distribution in categories of map A

Are these two maps really indifferent?

- Model 1 assumes that the region is homogeneous

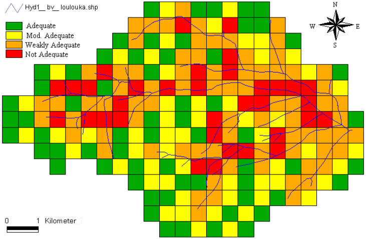

Figure : State 1

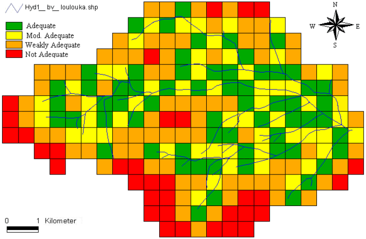

Figure : State 2

Are these two maps really indifferent?

- Model 1 assumes that the region is homogeneous

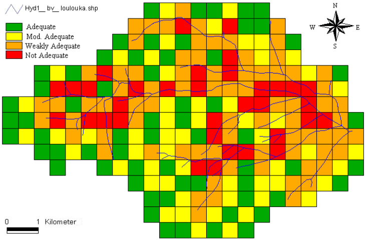

Figure : State 1

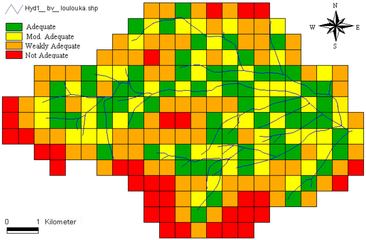

Figure : State 2

- Model 2 assumes that the region is composed of several homogeneous sub-regions (defined by a geographic aspect)

Model 2:

Taking geographic aspects into account

We generalize the expected utility model

$$A \succsim B \iff \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij}(A) u_j(E_{ij}) \geq \sum_{j=1}^{m} \sum_{i=1}^{n} x_{ij}(B) u_j(E_{ij})$$

where

- j is the homogeneous sub-region
- i is the category
- $x_{ij}(\cdot)$ is the proportion of the surface of region j assigned to category i
- $u_j(E_{ij})$ is the utility associated with the homogeneous sub-region j entirely assigned to category i

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Hypothesis

We assume that comparing maps only depends on the comparison of the area distributions of the homogeneous sub-regions: for all A, B, C, D, if $A \succsim B$, $x_j(A_j) = x_j(C_j)$ and $x_j(B_j) = x_j(D_j)$ for all j, then $C \succsim D$ with $x_j(\cdot) = (x_{1j}(\cdot), \ldots, x_{nj}(\cdot))$

Taking contiguity into account

Are these maps really indifferent?

Same distribution

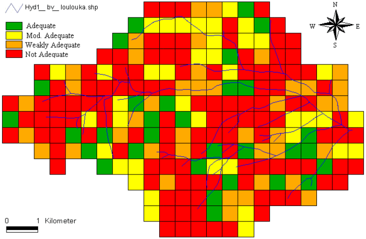

Figure : Scattered map

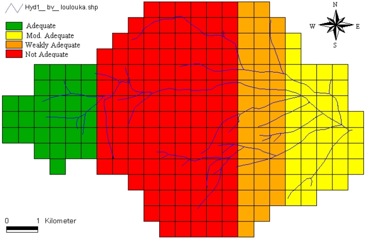

Figure : Clustered map

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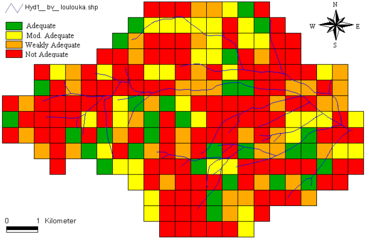

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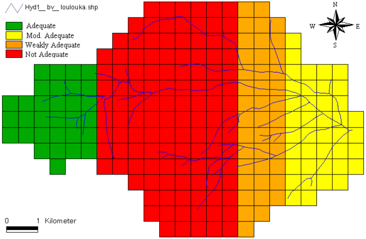

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⇒ Model based on the Choquet integral

The Choquet integral

No interaction:

$$\sum_{s \in S} m(\{s\}) u(\gamma(s))$$

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Pairwise interactions

$$\sum_{s \in S} m(\{s\})u(\gamma(s)) \quad + \quad \sum_{s,t \in S} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t)))$$

The Choquet integral

No interaction:

$$\sum_{s\in S} m(\{s\})u(\gamma(s))$$

Pairwise interactions + triple interactions

$$\sum_{s\in S} m(\{s\})u(\gamma(s)) \quad + \quad \sum_{s,t\in S} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t)))$$
$$+ \quad \sum_{r,s,t\in S} m(\{r,s,t\}) \min(u(\gamma(r)), u(\gamma(s)), u(\gamma(t)))$$

The Choquet integral

No interaction:

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Pairwise interactions + triple interactions + $\cdots$

$$\begin{aligned}\sum_{s \in S} m(\{s\}) u(\gamma(s)) \quad &+ \quad \sum_{s,t \in S} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t))) \\ &+ \quad \sum_{r,s,t \in S} m(\{r,s,t\}) \min(u(\gamma(r)), u(\gamma(s)), u(\gamma(t))) \\ &+ \quad \cdots\end{aligned}$$

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Pairwise interactions + triple interactions + $\cdots$

$$\begin{aligned} \sum_{s \in S} m(\{s\}) u(\gamma(s)) \quad &+ \sum_{s,t \in S} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t))) \\ &+ \sum_{r,s,t \in S} m(\{r,s,t\}) \min(u(\gamma(r)), u(\gamma(s)), u(\gamma(t))) \\ &+ \cdots \end{aligned}$$

Choquet integral:

$$C_m(u(\gamma(s_1)), \ldots, u(\gamma(s_N))) = \sum_{Y \subseteq \{1,\ldots,N\}} m(Y) \min_{i \in Y} u(\gamma(s_i))$$

The Choquet integral

No interaction:

$$\sum_{s \in S} m(\{s\}) u(\gamma(s))$$

Pairwise interactions + triple interactions + $\cdots$

$$\sum_{s \in S} m(\{s\}) u(\gamma(s)) + \sum_{s,t \in S} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t))) + \sum_{r,s,t \in S} m(\{r,s,t\}) \min(u(\gamma(r)), u(\gamma(s)), u(\gamma(t))) + \cdots$$

Choquet integral:

$$C_m(u(\gamma(s_1)), \ldots, u(\gamma(s_N))) = \sum_{Y \subseteq \{1,\ldots,N\}} m(Y) \min_{i \in Y} u(\gamma(s_i))$$

Interactions between K pixels $\Rightarrow$ K-additive Choquet integral

The Choquet integral is usually defined by a capacity...

Capacity

Let $I = \{1, \ldots, N\}$. A capacity μ over I is a function $\mu : 2^I \to [0, 1]$ s.t.

1. $\mu(\emptyset) = 0$
2. $\mu(I) = 1$
3. $\forall A, B \subseteq I, A \subseteq B \Rightarrow \mu(A) \leq \mu(B)$

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Möbius transform

Let μ be a capacity over $I = \{1, \ldots, N\}$. The Möbius transform of μ is

$$m(A) = \sum_{B \subseteq A} (-1)^{|A \setminus B|} \mu(B), \forall A \subseteq I$$

... but we are working with its Möbius transform

Choquet integral

Let $I = \{1, \ldots, N\}$ and $x := (x_1, \ldots, x_N) \in \mathbb{R}^N$

$$C_m(x) = \sum_{A \subseteq I} m(A) \bigwedge_{i \in A} x_i$$

... but we are working with its Möbius transform

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$$C_m(x) = \sum_{A \subseteq I} m(A) \bigwedge_{i \in A} x_i$$

k-additive capacity

Let $I = \{1, \ldots, N\}$. A capacity μ over I is k-additive $(0 < k \leq N)$ if $m_\mu(A) = 0 \;\; \forall A \subseteq I$ s.t. $|A| > k$ and if $\exists A \subseteq I$ s.t. $|A| = k$ and $m_\mu(A) \neq 0$

We set the contiguity structure

2 pixels are contiguous $\iff$ they have a common edge

We only consider pairwise interactions

We define

$$m(\{s\}) = \alpha$$
$$m(\{s,t\}) = \begin{cases} \beta & \text{if } s,t \text{ are contiguous} \\ 0 & \text{otherwise} \end{cases}$$

$\beta > 0 \Rightarrow$ advantage to contiguity
$\beta < 0 \Rightarrow$ disadvantage to contiguity
$\beta = 0 \Rightarrow$ contiguity has no influence

We impose 3 conditions on m

3 conditions (Chateauneuf and Jaffray (1989)):

1. $m(\emptyset) = 0$
2. $\sum_{R \subseteq S} m(R) = 1$
3. $\sum_{R \subseteq T, R \ni s} m(R) \geq 0, \ \forall R \subseteq S, \forall s \in S$

We rewrite the Choquet integral

$$C_m(u,A) = \sum_{s \in S} m(\{s\}) u(\gamma(s)) + \sum_{\substack{s,t \in S \\ s,t \text{ contiguous}}} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t)))$$

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$$= \alpha \sum_{s \in S} u(\gamma(s)) + \beta \left(\sum_{\substack{s,t \in S \\ s,t \text{ contiguous}}} \min(u(\gamma(s)), u(\gamma(t))) \right)$$

We rewrite the Choquet integral

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\begin{aligned}
C_m(u, A) &= \sum_{s \in S} m(\{s\}) u(\gamma(s)) + \sum_{\substack{s,t \in S \\ s,t \text{ contiguous}}} m(\{s, t\}) \min(u(\gamma(s)), u(\gamma(t))) \\
&= \alpha \sum_{s \in S} u(\gamma(s)) + \beta \left(\sum_{\substack{s,t \in S \\ s,t \text{ contiguous}}} \min(u(\gamma(s)), u(\gamma(t))) \right) \\
&= \alpha \sum_{i=1}^{n} n_i(A) u_i + \beta \sum_{i=1}^{n} m_i(A) u_i \\
&= \sum_{i=1}^{n} u_i (\alpha n_i(A) + \beta m_i(A))
\end{aligned}
$$

where $n_i(A) =$ number of pixels s s.t. $u(\gamma(s)) = u_i$ and $m_i(A) =$ number of contiguous pairs $\{s, t\}$ s.t. $\min(u(\gamma(s)), u(\gamma(t))) = u_i$

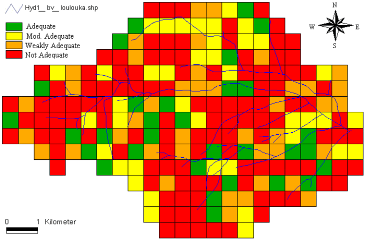


Figure : Scattered map

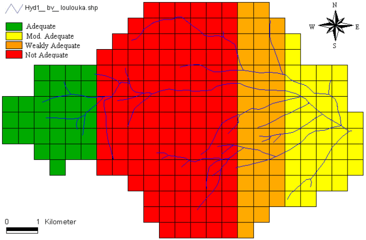


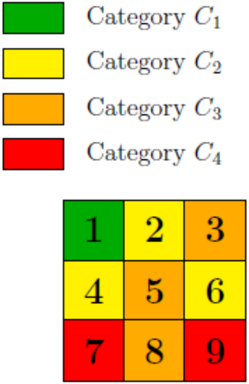
Figure : Clustered map

$$
\begin{aligned}
C_m(u, A) &= \alpha \sum_{i=1}^{n} n_i(A) u_i + \beta \sum_{i=1}^{n} m_i(A) u_i \\
&= \alpha(31u_1 + 40u_2 + 41u_3 + 117u_4) + \beta(5u_1 + 43u_2 + 58u_3 + 313u_4) \\
C_m(u, B) &= \alpha(31u_1 + 40u_2 + 41u_3 + 117u_4) + \beta(49u_1 + 75u_2 + 64u_3 + 231u_4)
\end{aligned}
$$

- $\beta > 0 \Rightarrow$ the clustered map is better
- $\beta < 0 \Rightarrow$ the scattered map is better

Question: what are the conditions underlying this model?

We represent contiguity by a graph



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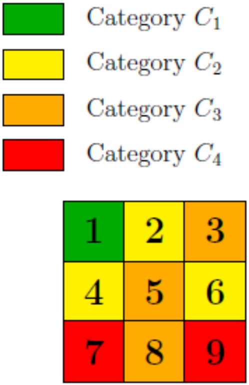

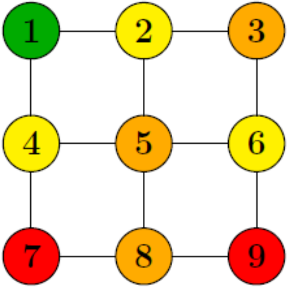

$$G = (I, E)$$

Wakker characterizes a "general" Choquet integral

Characterization (Wakker 1989)

Under some topological assumptions, the following two statements are equivalent:

(i) $\succsim$ can be represented by a Choquet integral

$$C_m(u(x_1), \dots, u(x_N)) = \sum_{Y \subseteq \{1,\dots,N\}} m(Y) \min_{i \in Y} u(x_i)$$

(ii) The binary relation $\succsim$ does not reveal comonotonic contradictory tradeoffs

We use the following notation

$I = \{1, \ldots, N\}$: set of criteria

$X^N = X \times \cdots \times X$: set of alternatives

$\succsim$: preference relation on X^N

$\succsim_0$: preference relation on X

Wakker works with comonotonic alternatives

Comonotonic alternatives

$x = (x_1, \ldots, x_N)$ and $y = (y_1, \ldots, y_N)$ are comonotonic if $\forall i, j \in \{1, \ldots, N\}$

$$x_i \succ_0 x_j \Rightarrow \text{ not } (y_j \succ_0 y_i)$$

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Students	Maths	Physics	Literature
A	18	16	10
B	10	12	18
C	14	15	15

We illustrate Wakker's axiom

Students	Maths	Physics	Literature
A	18	12	6
B	16	11	10
A'	14	12	6
B'	13	11	10
C	18	12	10
D	16	12	12
C'	14	12	10
D'	13	12	12

We illustrate Wakker's axiom

Students	Maths	Physics	Literature	Mean
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C	18	12	10	13.3
D	16	12	12	13.3
C'	14	12	10	12
D'	13	12	12	12.3

$A \prec B \quad A' \sim B' \quad C \sim D \quad C' \prec D'$

Wakker's axiom is based on the revelation of comonotonic contradictory tradeoffs

Comonotonic contradictory tradeoffs

$\succsim$ reveals comonotonic contradictory tradeoffs if $\exists x_{-i}\alpha, y_{-i}\beta, x_{-i}\gamma, y_{-i}\delta$ comonotonic such that

$$x_{-i}\alpha \precsim y_{-i}\beta \text{ and } x_{-i}\gamma \succsim y_{-i}\delta$$

and if $\exists v_{-j}\alpha, w_{-j}\beta, v_{-j}\gamma, w_{-j}\delta$ comonotonic such that

$$v_{-j}\alpha \succsim w_{-j}\beta \text{ and } v_{-j}\gamma \prec w_{-j}\delta$$

We illustrate Wakker's axiom

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$A \prec B \quad A' \sim B' \quad C \sim D \quad C' \prec D'$

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We obtain a particular Choquet integral

Characterization

Let $G = (I, E)$ be an interaction graph that does not contain any complete subgraph with 3 nodes or more. Under the same topological assumptions as Wakker's, the following two statements are equivalent:

(i) $\succsim$ can be represented by a 2-additive Choquet integral such that

$$m(\{s,t\}) = 0 \text{ if } \{s,t\} \notin E$$

(ii) The binary relation $\succsim$ does not reveal G-comonotonic contradictory tradeoffs

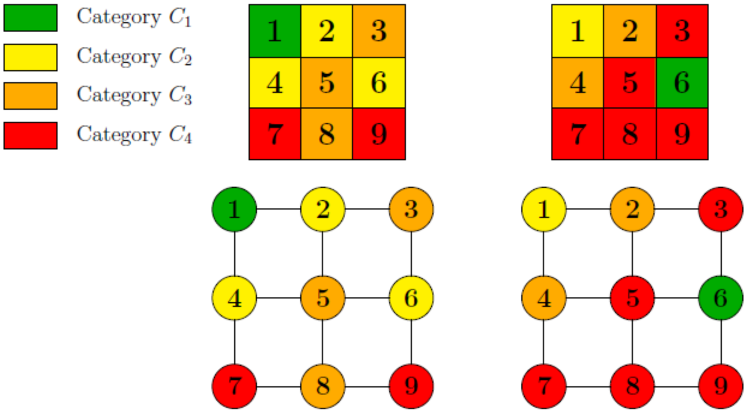
We work with G-comonotonic alternatives

G-Comonotonic alternatives

$x = (x_1, \dots, x_N)$ and $y = (y_1, \dots, y_N)$ are G-comonotonic if $\forall \{i, j\} \in E$

$$x_i \succ_0 x_j \Rightarrow \text{not } (y_j \succ_0 y_i)$$

Here are two G-comonotonic maps



Comonotonicity implies G-comonotonicity

Comonotonic $\Rightarrow$ G-comonotonic

Comonotonicity implies G-comonotonicity

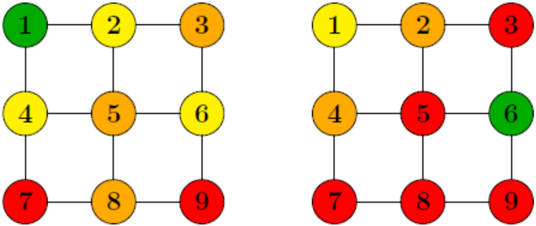
Comonotonic $\Rightarrow$ G-comonotonic

G-comonotonic $\not\Rightarrow$ comonotonic

Comonotonicity implies G-comonotonicity

Comonotonic $\Rightarrow$ G-comonotonic

G-comonotonic $\not\Rightarrow$ comonotonic



$x_1 \succ_0 x_6$ $\quad$ $y_6 \succ_0 y_1$

Our characterization is based on the revelation of G-comonotonic contradictory tradeoffs

Revelation of G-comonotonic contradictory tradeoffs

$\succsim$ reveals G-comonotonic contradictory tradeoffs if $\exists x_{-i}\alpha, y_{-i}\beta, x_{-i}\gamma, y_{-i}\delta$ G-comonotonic such that

$$x_{-i}\alpha \precsim y_{-i}\beta \text{ and } x_{-i}\gamma \succsim y_{-i}\delta$$

and if $\exists v_{-j}\alpha, w_{-j}\beta, v_{-j}\gamma, w_{-j}\delta$ G-comonotonic such that

$$v_{-j}\alpha \succsim w_{-j}\beta \text{ and } v_{-j}\gamma \prec w_{-j}\delta$$

Our axiom implies Wakker's axiom

Comonotonic $\Rightarrow$ G-comonotonic

Our axiom implies Wakker's axiom

Comonotonic $\Rightarrow$ G-comonotonic

Revelation of comonotonic contradictory tradeoffs $\Rightarrow$
revelation of G-comonotonic contradictory tradeoffs

Our axiom implies Wakker's axiom

Comonotonic $\Rightarrow$ G-comonotonic

Revelation of comonotonic contradictory tradeoffs $\Rightarrow$
revelation of G-comonotonic contradictory tradeoffs

Non revelation of G-comonotonic contradictory tradeoffs $\Rightarrow$
non revelation of comonotonic contradictory tradeoffs

We obtain a particular Choquet integral

Characterization

Let $G = (I, E)$ be an interaction graph. Under the same topological assumptions as Wakker's, the following two statements are equivalent:

(i) $\succsim$ can be represented by

$$C_m(u(x_1), \ldots, u(x_N)) = \sum_{Y \subseteq \{1,\ldots,N\}} m(Y) \min_{i \in Y} u(x_i)$$

and

$$m(Y) = 0 \text{ if } Y \text{ is not a complete subgraph}$$

(ii) The binary relation $\succsim$ does not reveal G-comonotonic contradictory tradeoffs

We obtain a particular Choquet integral

Characterization

Let $G = (I, E)$ be an interaction graph that does not contain any complete subgraph with 3 nodes or more. Under the same topological assumptions as Wakker's, the following two statements are equivalent:

(i) $\succsim$ can be represented by a 2-additive Choquet integral such that

$$m(\{s, t\}) = 0 \text{ if } s, t \text{ are not linked by an edge in } G$$

(ii) The binary relation $\succsim$ does not reveal G-comonotonic contradictory tradeoffs

We obtain a particular Choquet integral

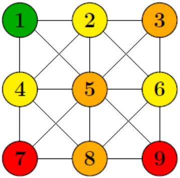
Characterization

Let $G = (I, E)$ be an interaction graph that does not contain any complete subgraph with 5 nodes or more. Under the same topological assumptions as Wakker's, the following two statements are equivalent:

(i) $\succsim$ can be represented by a 4-additive Choquet integral such that

$$m(Y) = 0 \text{ if } Y \text{ contains two nodes not linked by an edge in } G$$

(ii) The binary relation $\succsim$ does not reveal G-comonotonic contradictory tradeoffs



We obtain a particular Choquet integral

Characterization

Let $G = (I, E)$ be an interaction graph that does not contain any complete subgraph with $K + 1$ nodes or more. Under the same topological assumptions as Wakker's, the following two statements are equivalent:

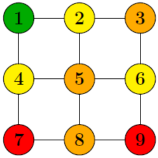
(i) $\succsim$ can be represented by a K-additive Choquet integral such that

$$m(Y) = 0 \text{ if } Y \text{ contains two nodes not linked by an edge in } G$$

(ii) The binary relation $\succsim$ does not reveal G-comonotonic contradictory tradeoffs

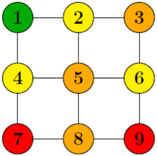
Going back to maps comparison

2 pixels are contiguous $\iff$ they have a common edge
We only consider pairwise interactions



Going back to maps comparison

2 pixels are contiguous $\iff$ they have a common edge
We only consider pairwise interactions



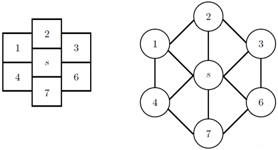
$$m(\{s\}) = \alpha$$

$$m(\{s,t\}) = \begin{cases} \beta & \text{if } s,t \text{ are contiguous} \\ 0 & \text{otherwise} \end{cases}$$

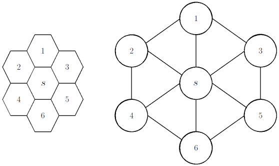
$$C_m(u,A) = \sum_{s\in S} m(\{s\})u(\gamma(s)) + \sum_{\substack{s,t\in S \\ s,t \text{ contiguous}}} m(\{s,t\}) \min(u(\gamma(s)), u(\gamma(t)))$$

$$= \sum_{i=1}^{n} u_i(\alpha n_i(A) + \beta m_i(A))$$

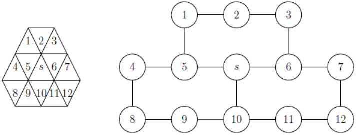
Going back to maps comparison



3-additive Choquet integral



3-additive Choquet integral



2-additive Choquet integral

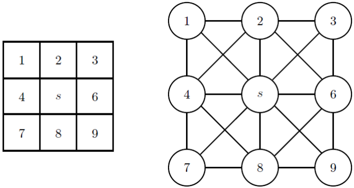
Going back to maps comparison

Four color theorem: with four colors, it is possible to color any connected map in such a way that two regions sharing a common boundary (not reduced to a single point) do not share the same color

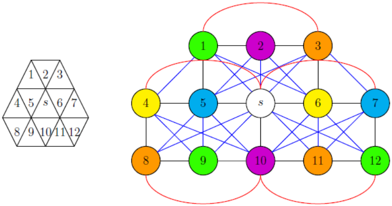
$\Rightarrow$ any connected map has no complete subgraph with five nodes

$\Rightarrow$ 4-additive Choquet integral

We can consider more pairwise interactions



4-additive Choquet integral

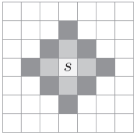


6-additive Choquet integral

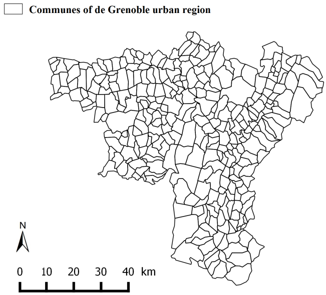
We can consider more pairwise interactions

$$\begin{aligned} C_m(u, A) = \alpha \sum_{s \in S} u(\gamma(s)) \quad & + \quad \beta_1 \sum_{\{s,t\}:d_1(s,t)=1} \min(u(\gamma(s)), u(\gamma(t))) \\ & + \quad \beta_2 \sum_{\{s,t\}:d_1(s,t)=2} \min(u(\gamma(s)), u(\gamma(t))) \\ & + \quad \cdots \\ & + \quad \beta_k \sum_{\{s,t\}:d_1(s,t)=k} \min(u(\gamma(s)), u(\gamma(t))), \end{aligned}$$

with $d_1(s,t)$ the L_1-distance defined by $d_1(s,t) = |s_1 - t_1| + |s_2 - t_2|$, $s = (s_1, s_2), t = (t_1, t_2)$



We can consider an irregular sub-division of the territory



$$m(\{s\}) = \alpha$$
$$m(\{s,t\}) = \begin{cases} \beta & \text{if } s,t \text{ are contiguous} \\ 0 & \text{otherwise} \end{cases}$$

α can depend on the surface area of s
β can depend on the length of the common border between s and t

Conclusion

- Our characterization depends on the interaction structure

Conclusion

- Our characterization depends on the interaction structure
- Our interactions are well-defined in terms of contiguity

Favored configuration with $\beta > 0$

0	0	1	1	2	2

Favored configuration with $\beta < 0$

2	0	2	0	1	1

Thank you for your attention

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POLYTECH MONS

Modelling contiguity in spatial decision contexts

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UMONS - Faculty of Engineering

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